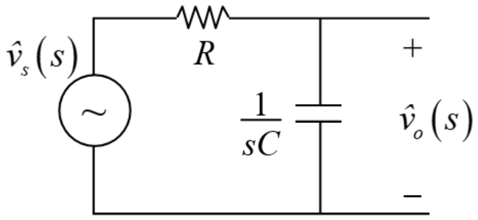
Chap 6 Linear Circuits with Operational Amplifier

6.1 Fundamental concepts of filters

- First-order low pass-filter implemented by an RC cirduit:



- Assume the voltage source is

(6.1-1) $v_s(t)=V_s cos(\omega t)$ or $\hat{v}_s(s)=V_s\frac{s}{s^2+\omega^2}$.

Then, the output voltage is

(6.1-2) $$\hat{v}_o(s)=\frac{\frac{1}{sC}}{R+\frac{1}{sC}}\hat{v}_s(s)=\underbrace{\frac{1}{1+sRC}}_{H(s)}\cdot\hat{v}_s(s)=H(s)\frac{V_s\ s}{s^2+\omega^2}$$

- As $t\to\infty$, let $s=j\omega$ and obtain $H(j\omega)=\frac{1}{1+j\omega RC}$, or expressed as

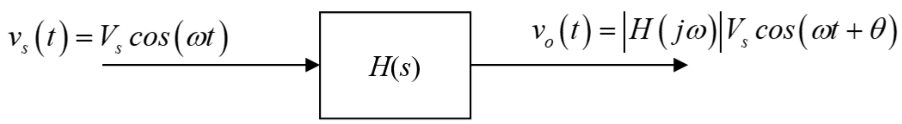
(6.1-3) $$H(j\omega)=|H(j\omega)|e^{j\angle H(j\omega)}=\frac{1}{\sqrt{1+(\omega RC)^2}}e^{j\theta}$$

where $|H(j\omega)|=\frac{1}{\sqrt{1+(\omega RC)^2}}$ and $\theta=\angle H(j\omega)=-tan^{-1}(\omega RC)$.

- If $v_s(t)=V_s cos(\omega t)$ then the output voltage is

(6.1-4) $$v_o(t)=|H(j\omega)|V_s cos(\omega t+\theta)=V_0 cos(\omega t+\theta)$$

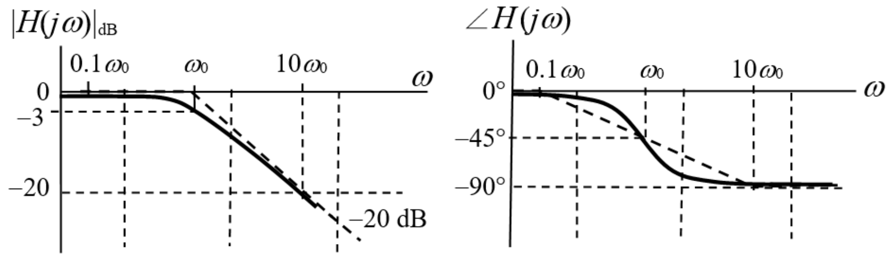
where $V_0=|H(j\omega)|V_s$.



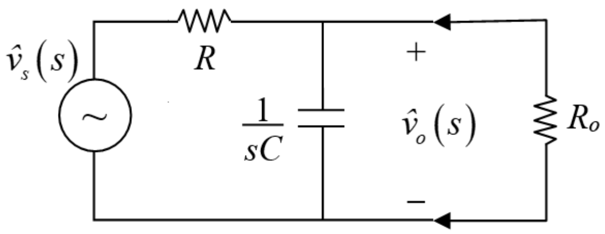
- It is easy to check that $\left|H(j\omega)\right| = \dfrac{1}{\sqrt{1+(\omega RC)^2}} < 1$, i.e., $V_0 < V_s$ and

(6.1-5) $$\left|H(j\omega)\right| = \begin{cases} 1^- & \omega \to 0 \\ 1/\sqrt{2} & \omega = \omega_0 = 1/RC \\ 0^+ & \omega \to \infty \end{cases}$$

The output voltage will be decresed while ω is incresed.



- To achieve the output voltage, we often connect a receiver with equivalent resistance R_o, shown as below:



Then, the output voltage is changed as $\hat{v}_o(s) = H_{R_o}(s)\hat{v}_s(s)$ or

(6.1-6) $$\hat{v}_o(s) = \frac{\frac{1}{sC} \,||\, R_o}{R + \frac{1}{sC} \,||\, R_o}\hat{v}_s(s) = \underbrace{\frac{r}{r(sRC)+1}}_{H_{R_o}(s)} \cdot \frac{V_s\, s}{s^2+\omega^2}$$

where $r = \dfrac{R_o}{R+R_o} < 1$.

- It is easy to check that $\left|H_{R_o}(j\omega)\right| = \dfrac{r}{\sqrt{1+r^2(\omega RC)^2}} < 1$, i.e., $V_0 < V_s$ and

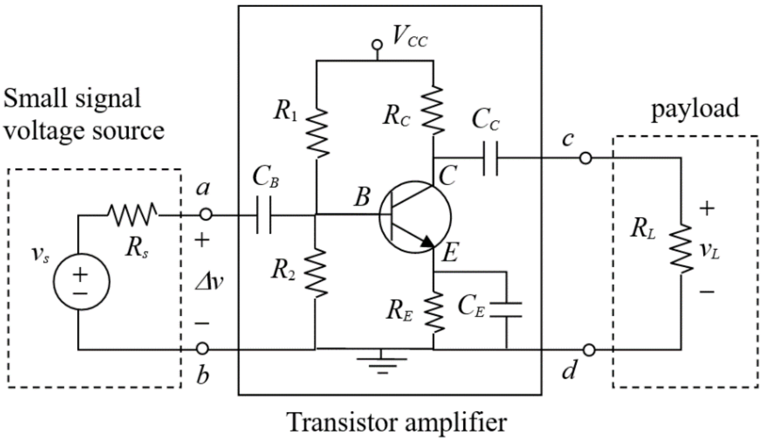
(6.1-7) $$\left|H_{R_0}(j\omega)\right| = \begin{cases} r^- & \omega \to 0 \\ r/\sqrt{2} & \omega = \omega_0 = 1/rRC \\ 0^+ & \omega \to \infty \end{cases}$$

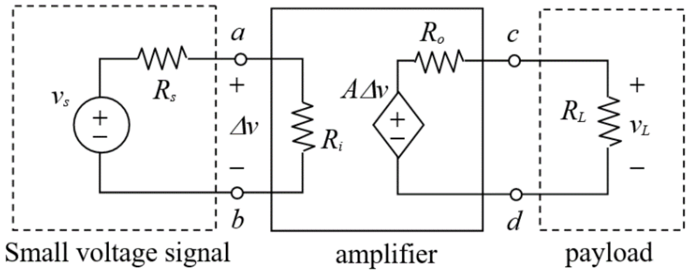
- Note that if $R_o \to \infty$, then $r \to 1$ and $H_{R_o}(j\omega) \approx H(j\omega)$.

  However, it is impossible to use $R_o \to \infty$ since the cureent through R_o will be very small, so is the output power.

6.2 Ideal Operational Amplifier (OPAmp)

- Voltage Amplifier using transistor

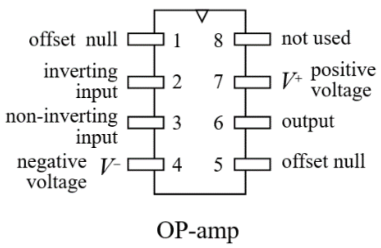


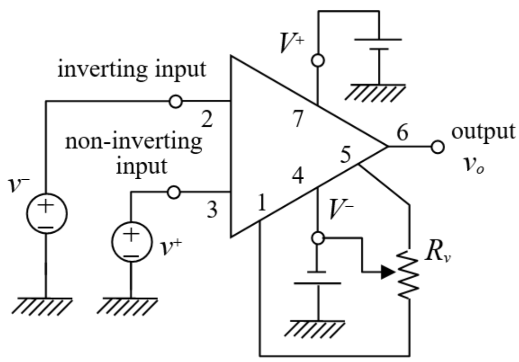


(6.2-1) $$v_L(t) = \frac{R_L}{R_o + R_L} A\Delta v(t) = \left(\frac{R_L}{R_o + R_L}\right)\left(\frac{R_i}{R_i + R_s}\right) A v_s(t)$$

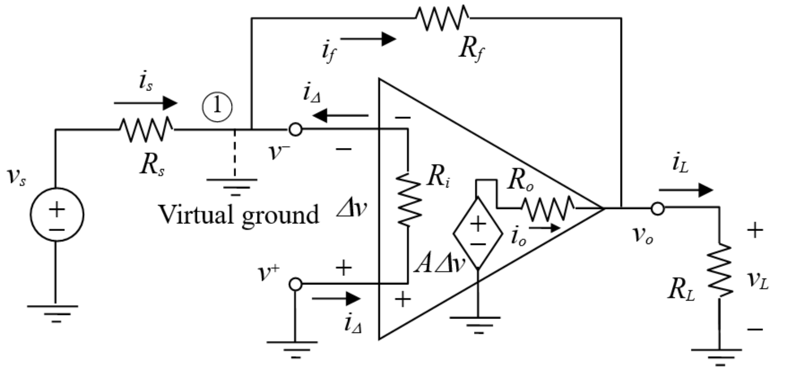
If $R_i \to \infty$ and $R_o \approx 0$, then $v_L(t) = A v_s(t)$, not affected by R_s and R_L.

- OPAmp





- Importent properties

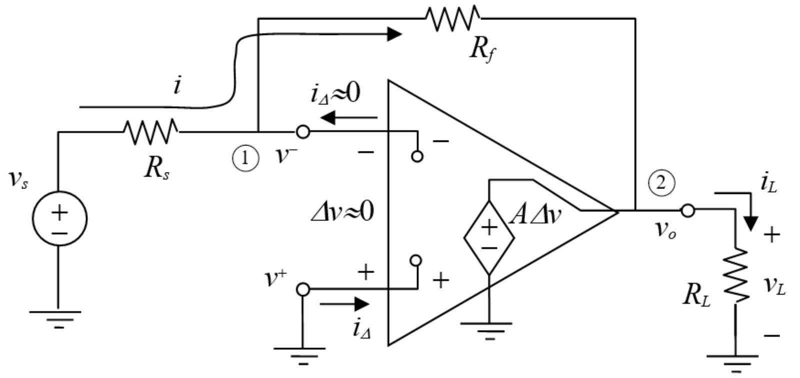


(6.2-2) $R_i \to \infty$, $R_o \approx 0$, , $A \to \infty$, ,

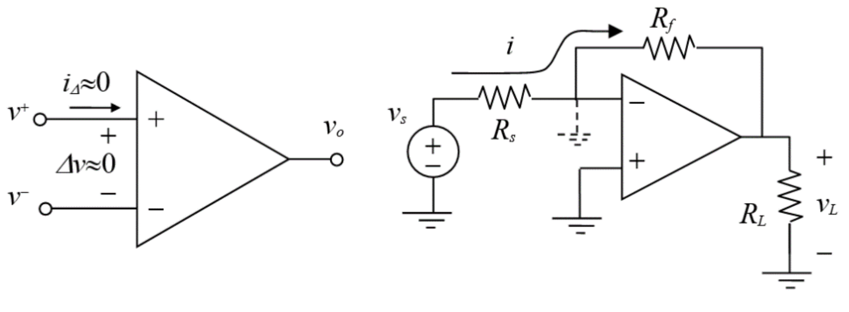
(6.2-3) $v_o(t) = A\Delta v(t)$

It implies $\Delta v(t) \approx 0$ and $i_\Delta(t) \to 0$.

In general, we say that the node ① is a virtual ground.



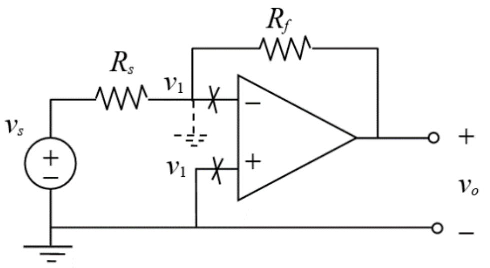
- Equivalent circuit



(6.2-4) $i(t) = \dfrac{v_s(t)}{R_s} = -\dfrac{v_L(t)}{R_f} \Rightarrow v_L(t) = -\dfrac{R_f}{R_s} v_s(t)$

That means the gain $-\dfrac{R_f}{R_s}$ is independent to the payload R_L.

- Inverting amplifier

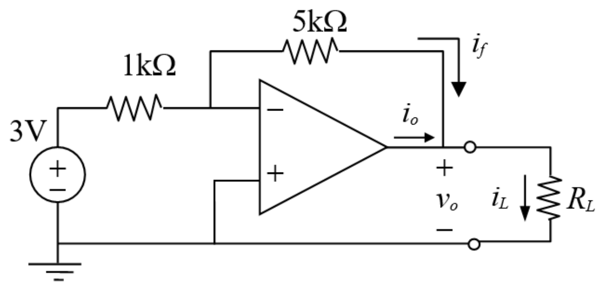


(6.2-5) $$\frac{v_1(t)-v_s(t)}{R_s}+\frac{v_1(t)-v_o(t)}{R_f}=0 \Rightarrow v_o(t)=-\frac{R_f}{R_s}v_s(t)$$

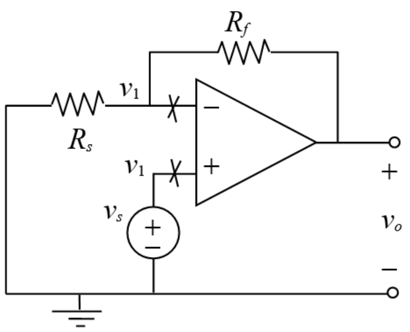
(6.2-6) $$A_v=\frac{v_o(t)}{v_s(t)}=-\frac{R_f}{R_s}$$

The voltage gain is negative and called the inverting amplifier.

Example: If $R_L=1\ \text{k}\Omega$, what is $i_L(t)$?



- Non-inverting amplifier



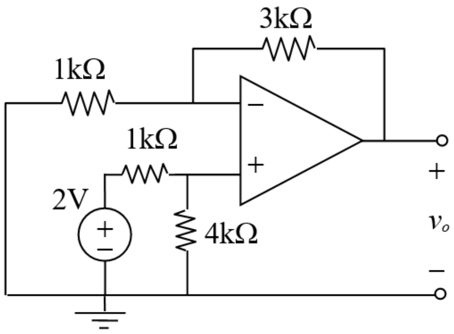
(6.2-7) $$\frac{v_1(t)}{R_s}+\frac{v_1(t)-v_o(t)}{R_f}=0 \Rightarrow v_o(t)=\left(1+\frac{R_f}{R_s}\right)v_s(t)$$

(6.2-8) $$A_v=\frac{v_o(t)}{v_s(t)}=1+\frac{R_f}{R_s}\geq 1$$

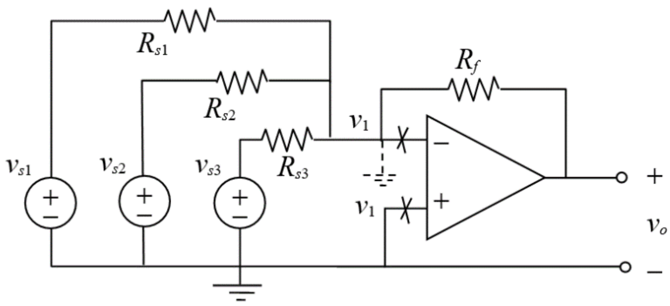
The voltage gain is positive and called the non-inverting amplifier.

It can be used as a buffer, $v_o(t)=v_s(t)$, by setting R_f=0 or R_s=∞.

Example: Determine $v_o(t)$.



- Addition

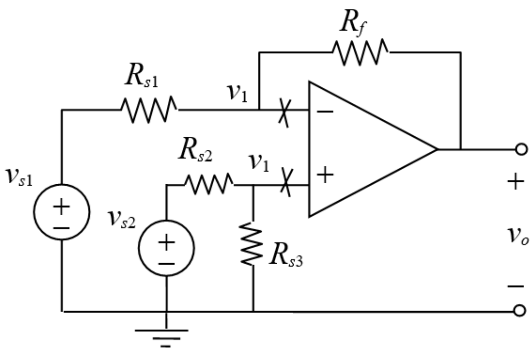


(6.2-9) $$\frac{v_1(t)-v_{s1}(t)}{R_{s1}}+\frac{v_1(t)-v_{s2}(t)}{R_{s2}}+\frac{v_1(t)-v_{s3}(t)}{R_{s3}}+\frac{v_1(t)-v_o(t)}{R_f}=0$$

(6.2-10) $$v_o(t)=-\left(\frac{R_f}{R_{s1}}v_{s1}(t)+\frac{R_f}{R_{s2}}v_{s2}(t)+\frac{R_f}{R_{s3}}v_{s3}(t)\right)$$

It is an inverting addition.

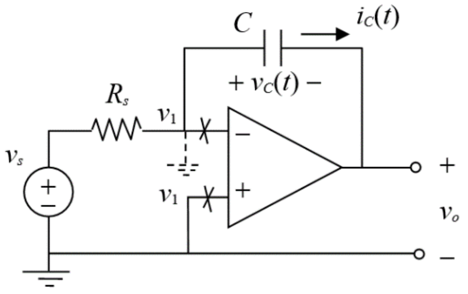
- Subtraction



(6.2-11) $$\frac{v_1(t)-v_{s1}(t)}{R_{s1}}+\frac{v_1(t)-v_o(t)}{R_f}=0\,,\quad v_1(t)=\frac{R_{s3}}{R_{s2}+R_{s3}}v_{s2}(t)$$

(6.2-12) $$v_o(t)=\frac{R_{s3}(R_f+R_{s1})}{R_{s1}(R_{s3}+R_{s2})}v_{s2}(t)-\frac{R_f}{R_{s1}}v_{s1}(t)$$

- Integrator



(6.2-13) $i_C(t) = C\dfrac{dv_C(t)}{dt} = C\dfrac{d}{dt}(v_1(t) - v_o(t))$

(6.2-14) $\dfrac{v_1(t) - v_s(t)}{R_s} + i_C(t) = \dfrac{v_1(t) - v_s(t)}{R_s} + C\dfrac{d}{dt}(v_1(t) - v_o(t)) = 0$

Since $v_1(t)$=0, we have

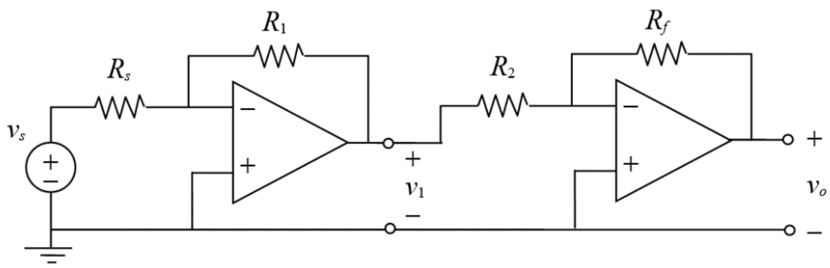
(6.2-15) $\dfrac{dv_o(t)}{dt} = -\dfrac{v_s(t)}{R_s C}$

(6.2-16) $v_o(t) = -\dfrac{1}{R_s C}\int_0^t v_s(\tau)d\,\tau + v_o(0)$

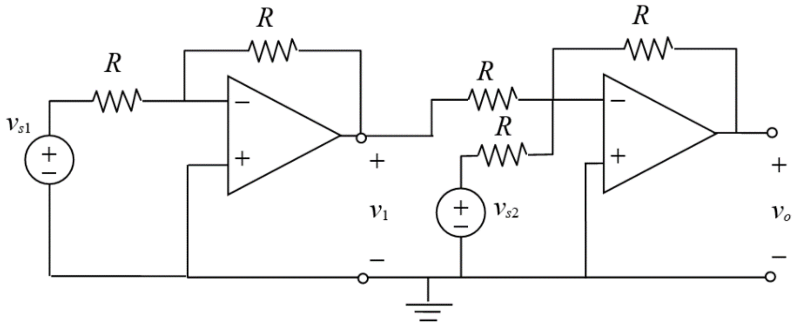
It is an inverting integrator.

6.3 RLC circuits with OPAmps

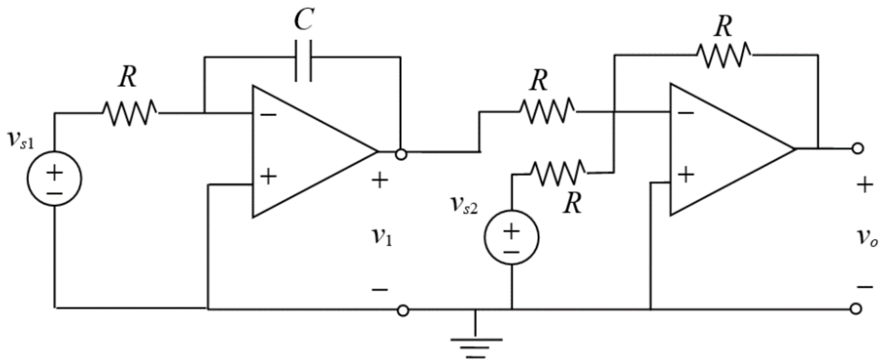
- Example-1



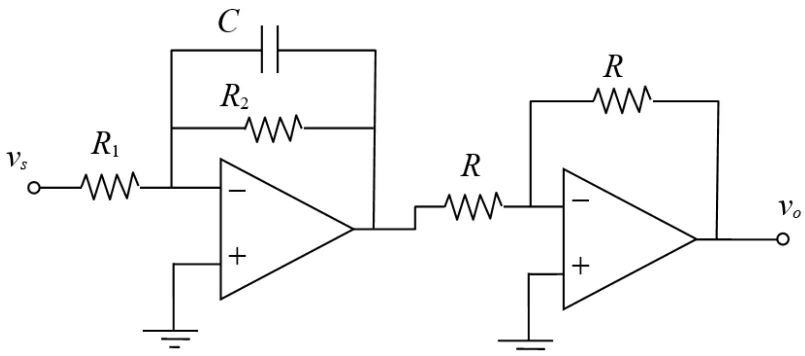
- Example-2



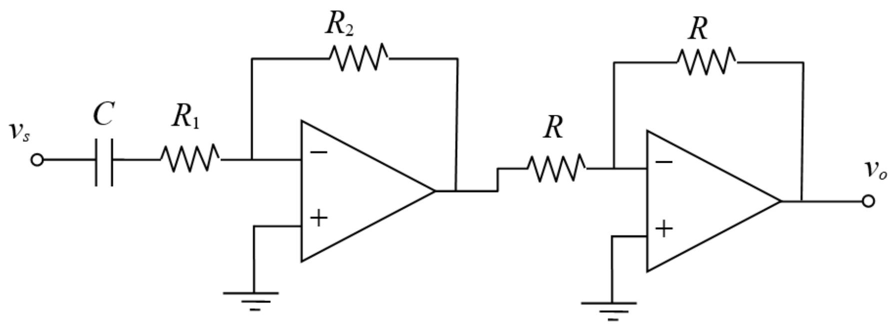
- Example-3



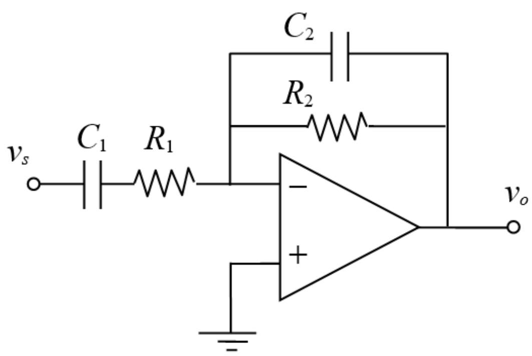
- Example-4



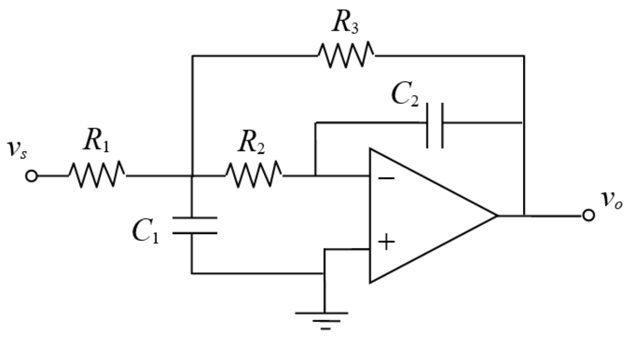
- Example-5



- Example-6



- Example-7



- Example-8